Case Study on the Integration of History of Mathematics into Advanced Mathematics Teaching

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Keywords: Advanced mathematics, History of mathematics, Mathematics thought

Abstract: Advanced mathematics is a basic compulsory course for higher education, and it is also the most important and practical tool course. The establishment of advanced mathematics courses not only lays a theoretical foundation for subsequent professional courses, but also cultivates students' mathematics literacy so that students can view problems from a mathematical perspective and form rational thinking and logical reasoning capabilities. This article gives some teaching cases that integrate the history of mathematics in the process of higher mathematics teaching. By digging the source of mathematics ideas and enriching the teaching content, we enhance students' interest in learning mathematics and the ability to solve problems in applied mathematics.

1. Introduction

Mathematics is a kind of human culture with special functions. It contains the humanistic spirit and embodies the human spirit of exploration, truth-seeking, cooperation and dedication. Mathematics is also a unique method of thinking, which has become the core driving force for the development of high-end scientific and technological innovation [1]. With the improvement of the status of mathematics, the status and function of mathematics education have also received more and more attentions. As a public basic course of university mathematics, advanced mathematics is an important foundation and tool for the successor main courses of many majors. It is also an important carrier for cultivating students' innovative ability and promoting the process of quality education. However, in the process of advanced mathematics teaching, teachers generally only simply teach mathematics theories and methods. This teaching method can help students easily cope with exams, but students know very little about the significance of learning advanced mathematics.

As an interdisciplinary subject between mathematics and history, the history of mathematics is a science that studies the occurrence, development and laws of mathematical. It not only traces the evolution and development process of the content, ideas and methods of mathematics, but also explores the various factors that affect the development of mathematics. In short, incorporating the history of mathematics into the advanced mathematics curriculum can not only make students feel the process of mathematics creation, but also help them grasp the context of mathematical knowledge, understand the cultural value and application value of mathematics, and cultivate their scientific and rigorous spirit.

  DOI: 10.25236/isaete.2021.043

2. The Educational Value of the History of Mathematics

In recent years, the relationship between the history of mathematics and mathematics education and the application of the history of mathematics in mathematics teaching have received increasing attention from the mathematics education field. The following will discuss the educational value of the history of mathematics in mathematics teaching from four aspects[2].

Firstly, integrating the history of mathematics into mathematics teaching can stimulate students' interest in learning and form a positive attitude towards mathematics. Since advanced mathematics has its high degree of abstraction and rigorous logic, college students often become afraid of it when they talk about “numeracy”. Therefore, how to improve the enthusiasm and initiative of college students in classroom learning has always been a difficult problem in higher mathematics education.

The history of mathematics has rich and colourful connotations. Lively and interesting stories, wisdom that travels through time and space, problems that are close to life, and pleasing art can fully arouse students' enthusiasm and enable them to form good study attitudes.

Secondly, integrating the history of mathematics into mathematics teaching can strengthen students' patriotism education, increase their national pride, and cultivate a sense of social responsibility. The history of mathematics has strong era and national characteristics. Integrating the history of mathematics into daily teaching, introducing students the ancient Chinese mathematics achievements of world significance (such as Liu Hui’s circumcision technique), and making them understand the great wisdom and creativity of their ancestors can enhance students’ national pride and inspire their Patriotic passion.

Thirdly, integrating the history of mathematics into mathematics teaching can enable students to achieve the power of role models, exercise their strong will, and cultivate good psychological qualities. In the history of mathematics, many mathematicians are models of success in adversity and never giving up. Anaxagoras is stuck in a difficult situation but he continues to explore, Sophie Germain is still studying diligently on a winter night when the ink freezes... The deeds of these outstanding historical figures can be used to motivate students to study hard and persevere.

3. Cases of Integrating History of Mathematics into the Course Teaching Process

3.1 Grandi “Made out of Nothing” and the Definition of Series Convergence

On the basis of studying a series of numbers, physicists and mathematicians always want to study the sum of an infinite series, so they have the concept of series, but does this “sum” exist? [2] The most confusing series in the history of mathematics is the alternating series $\sum_{n=1}^{\infty}(-1)^{n-1}=1-1+1-1+\cdots$ From the 17th century to the 18th century, many first-class mathematicians believed that this series should be harmonious. In 1703, the Italian mathematician G. Grandi used the function expansion

$$\frac{1}{1+x}=1-x+x^2-x^3+\cdots \quad (1)$$

He let $x=1$ (Now we know that this is wrong) and got $1-1+1-1+1-1+\cdots=\frac{1}{2}$.Then From the other perspective, $1-1+1-1+1-1+\cdots=(1-1)+(1-1)+\cdots=0$. Therefore, G. Grandi said that the world can indeed be created from nothing. Grandi called this result as “out of nothing”. Leibniz wrote to the German mathematician C.Wolf in 1715 and pointed that Grandi's result was correct.

In that era, the concept of convergence and divergence of infinite series had not yet entered the field of vision of mathematicians, so many mistakes are appeared, and some scholars even gave absurd explanations. It was not until the beginning of the century that mathematicians began to consider the rigorization of analysis, which introduced the concept of series convergence and divergence.

By introducing such historical facts, it is helpful for teachers to predict and determine the cognitive impairment of students and error-prone points [3], so that in the following teaching, teachers will emphasize the two pionts:(1) The basic idea of studying the constant term series is to “consider the limit of the partial sum of the sequence of numbers”, (2) we cannot directly extend the algorithm of finite numbers to the summation of constant term series. At the same time, the mistakes made by mathematicians due to historical limitations can make students subconsciously clarify that the constant series is a brand new problem that needs to be treated with caution, so as to cultivate students' qualities of adhering to the truth and being positive.

3.2 Liu Hui's “Circle-Cutting Technique “and the Limit Problem

In advanced mathematics teaching, the concept of limit has a complex logical structure, and the $\varepsilon - N$ “language” which is used to describe the limits of the sequence is highly abstract. It is difficult for students to construct the concept of limit in their own cognitive system. Students are often passive in learning the concept of limit. In teaching, if teachers can find an entry point from a historical perspective and guide students to actively explore the essence of the limit thinking method, it will help students better understand the concept of limit.

The ancient Chinese mathematician Liu Hui created the famous “circle-cutting technique” in “Notes on Nine Chapters of Arithmetic”. He used the relationship of area and the limit thoughts in the figure to get the area formula of the circle [4].

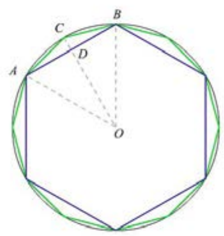

Fig.1 The Graph for Circle-Cutting Technique

In the figure above, let $AB = a_n, AC = a_{2n}$, The perimeter of n-sided Regular polygon inscribed in the circle $l_n = n \cdot a_n$,The area of the 2n-sided regular polygon inscribed in the circle can be descried by

$$S_{2n} = 2n \cdot S_{\Delta AOC} = 2n \cdot (\frac{1}{2} AD \cdot OC) = 2n \cdot (\frac{1}{4} AB \cdot OC) = 2n \cdot (\frac{1}{4} a_n \cdot r) = \frac{1}{2} l_n r \quad (2)$$

Suppose the area of the circle be S, then it can be shown that $S > S_{2n} > S_n$ and further $S > \cdots > S_{4n} > S_{2n} > S_n$. Liu Hui's commentary said, “If you cut it thinly, you lose less. If you cut it and cut it, so that it can't be cut, it merges with the circle without losing anything.” That means if the finer the circle is divided, then the difference between the circumference of a regular polygon and the circumference is smaller and the area of the inscribed regular polygon is closer to the area of the circle.

Then it can be shown that $S=\lim_{n\to\infty}S_{2n}=\lim_{n\to\infty}\frac{1}{2}l_n r=\frac{1}{2}lr$.

The language description of the limit of the sequence of numbers is given below: For a arbitrarily small positive number $\varepsilon>0$, there always exists N (only dependent on ε), when $n>N(\varepsilon)$, such that $|S-S_{2n}|=\left|S-\frac{1}{2}l_n r\right|<\varepsilon$.

In short, Liu Hui's “circle-cutting technique “ implies a simple infinite thought and limit thought. Choosing such materials for the history of mathematics can make students understand the great wisdom and creation of ancient Chinese, and encourage students to establish “cultural confidence”. In addition, by introducing the calculus thought in Liu Hui's “Circle-cutting technique”, students can understand the definition of limit well in essence, and it make abstract language come alive.

3.3 Newton and the Concept of “Derivative”

In the teaching for advanced mathematics, teachers give the concept of derivatives: if $y=f(x)$ has the instantaneous rate of change $\lim_{\Delta x\to 0}\frac{\Delta y}{\Delta x}=\lim_{\Delta x\to 0}\frac{f(x_0+\Delta x)-f(x_0)}{\Delta x}$ at the point $x=x_0$, we call it the derivative of the function $y=f(x)$ at the point $x=x_0$ and we denote it as $f'(x_0)$, where $f'(x_0)=\lim_{\Delta x\to 0}\frac{\Delta y}{\Delta x}=\lim_{\Delta x\to 0}\frac{f(x_0+\Delta x)-f(x_0)}{\Delta x}$. Since the derivative is defined by the limit, and the specific definition of the limit is too obscure and difficult to understand, students cannot understand the concept of the derivative well. If teachers can find an entry point from a historical perspective in teaching and guide students to actively explore the definition of the derivative , it will help students to deepen their understanding for derivatives.

Obviously, the derivative in the definition refers the instantaneous speed, so how to find the instantaneous speed? [5] This used to be a problem that plagued many mathematicians at the beginning of the century. Many scientists tried to solve it. At that moment Newton created the derivative using the following method: A free-falling object has the distance of descent $S(t)=\frac{1}{2}gt^2$ during t times where g means the acceleration of gravity. From time t_0 to time $t_0+\Delta t$, the distance for the object falls is $\Delta S=S(t_0+\Delta t)-S(t_0)=\frac{1}{2}g[2t_0\Delta t+(\Delta t)^2]$ and the average speed is $\frac{\Delta S}{\Delta t}=gt_0+\frac{1}{2}g\Delta t$. At that time, Newton believed that the instantaneous velocity of an object at time t_0 is gt_0 (that is the derivative of $S(t)$ at time t_0). At this time, some students may ask a question: why can he drop the $\frac{1}{2}g\Delta t$ in the formula? In fact, at that moment in history, the focus of British Archbishop G. Berkeley’s attack on Newton was why Newton drop the $\frac{1}{2}g\Delta t$ but only maintain gt_0 in the formula $\frac{\Delta S}{\Delta t}=gt_0+\frac{1}{2}g\Delta t$.

Newton’s derivative method is to divide $\frac{\Delta S}{\Delta t}$ into two parts, one of which is not related to Δt (as a derivative), and the other is related to Δt which is about the infinitesimal amount of Δt and can be discarded. Although this definition is not rigorous, the conclusion is correct.

Through the introduction of the above history of mathematics in classroom teaching, on the one hand, it deepens students' understanding of the concept of derivatives, and on the other hand, it allows students to understand that the generation and development of calculus knowledge is also a long and rugged road. We must learn those people who work hard for the true knowledge of mathematics without fear of hardship in daily study and work.

4. Conclusion

Incorporating appropriate historical materials of mathematics into the teaching process in the higher education process is of great significance for improving students' interest in learning, enhancing the connotation of mathematical activities, and improving the quality of mathematics teaching and this is also well adapted to the demand for the cultivation of innovative and applied talents at the current stage of higher education.

However, due to a lot of teaching contents in advanced mathematics and the limited amount of class hours, how to carry out student-centered teaching containing the history of mathematics under limited conditions has raised higher requirements for the teaching design of advanced mathematics, and this requires mathematics teachers to must be very familiar with the content of the history of mathematics, and can process historical materials according to the actual course and the subjects of the course, and finally show the theory, methods and philosophy of mathematics appropriately through the interesting classroom teaching.

Acknowledgement

This research was supported by the Pre-research project for major projects of Tianjin University of Finance and Economics Pearl River College called" Ideological and Political Research on College Mathematics Curriculum with OBE Educational Concepts from the Perspective of HPM " (Project Number: ZJZD21-10).

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